# Agribusiness Analysis and Forecasting

## Autoregressive Process, Part I

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# Autoregressive Process (AR)

- An *autoregressive* (AR) time series model amounts to forecasting a variable using only its own past values.
- We are going to focus on the application and less on the estimation calculations because AR models can be simply estimated using *OLS*.
- *Simetar* estimates *AR* models easily with a menu and provides forecasts of the time series model.

# AR Process

- *AR* is a forecasting methodology ideal for variables without clear relationships to other variables in the sense of a structural model.
- An AR process in the simplest form is a regression model such as:

$$Y_t = f(Y_{t-1}, Y_{t-2}, Y_{t-3}, ...)$$

- Notice there are no structural variables, just lags of the variable itself.

# AR Process

General steps for applying an autoregressive model are:

1. Graph the data series to see what patterns are present.
2. Test data for *stationarity* with Dickey-Fuller (D-F) tests.
   - If original series is not stationary then difference it until it is.
   - Number of Differences (p) to make a series stationary is determined using the D-F Test.
3. Use the stationary (differenced) data series to determine the number of Lags that best forecasts the historical period.
   - Use the Schwarz Criteria (SIC), autocorrelation table, or partial-autocorrelation table to determine the best number of lags (q) to include when estimating the model.
4. Estimate the $AR(p, q)$ Model with $OLS$ and make recursive forecasts.

# Stationarity

A series is *covariance stationary* if the mean and variability is constant, i.e., the same for the future as for the past, in other words.

- $E(Y_t) = E(Y_{t-1}) = \mu$
- $\sigma^2_{T+i} = \sigma^2_{Historical} < \infty$
- $Cov(Y_t, Y_{t-k}) = \gamma_k$ and does not depend on time.
- This is a crucial assumption because if $\sigma^2$ depends on $t$, then forecast variance will explode over time.

# Step to Insure the Data are Stationary

- Take differences of the data to make it stationary.
- The first difference of the raw data in $Y$ is

$$D_{1,t} = Y_t - Y_{t-1}$$

- Calculate the second difference of $Y$ using the first difference ($D_{1,t}$ ) or

$$D_{2,t} = D_{1,t} - D_{1,t-1}$$

- stop differencing data when series is stationary.

# Make Data Series Stationary

Example Difference table for a time series data set

| t | $Y$ | $D_1$ | $D_2$ |
|---|---|---|---|
| 1 | 71.06 | | |
| 2 | 71.47 | 0.41 | |
| 3 | 70.06 | -1.41 | -1.82 |
| 4 | 70.31 | 0.25 | 1.86 |

# Test for Stationarity

Dickey-Fuller Test for stationarity

- First D-F test: Are original data stationary?

$$D_{1,t} = \alpha + \beta Y_{t-1}$$

- $H_0$: the data are non-stationary
- Parameters can be estimated using OLS
- D-F Test statistic is the $t$ statistic on $\beta$.
- If $t$ is less than the critical value of -2.9 (more negative), reject $H_0$ at the 5% level.
- For instance, if you get a D-F statistic of -3.2, which is more negative than -2.9, then *independent* series are stationary.

# Next Level of Testing for Stationarity

- Second D-F Test: Testing for stationarity of the $D_{1,t}$ series with the Second D-F Test.
- Here we are testing if the $Y$ series will be stationary after only one differencing
  - So we are asking if the $D_{1,t}$ series is stationary.
- Estimate regression for

$$D_{2,t} = \alpha + \beta D_{1,t-1}$$

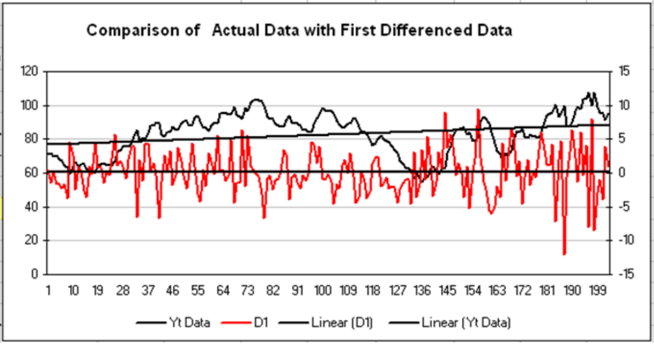
- $t$ statistic on slope $\beta$ is the second D-F test statistic.
- Check if the $t$ statistic is more negative than -2.90.

# Test for Stationarity

- Estimate regression for: $D_{1,t} = \alpha + \beta Y_{t-1}$.
- D-F is -1.868. You can see it is the $t$ statistic for the $\beta$ on the original series.

**OLS Regression Statistics**

| | | |
|---|---|---|
| F-test | 3.489 | Prob(F) |
| MSE $^{1/2}$ | 12.364 | CV Regr |
| $R^2$ | 0.017 | Durbin-W |
| RBar $^2$ | 0.012 | Rho |
| Akaike Inf | 5.030 | Goldfeld- |
| Schwarz I | 5.046 | |
| 95% | Intercept | Yt |
| Beta | 82.867 | -0.492 |
| S.E. | 0.868 | 0.263 |
| t-test | 95.426 | -1.868 |
| Prob(t) | 0.000 | 0.063 |
| Elasticity at Mean | | -0.001 |
| Variance Inflation F | NA | |
| Partial Correlation | NA | |
| Semipartial Correlat | NA | |
| Restriction | | |

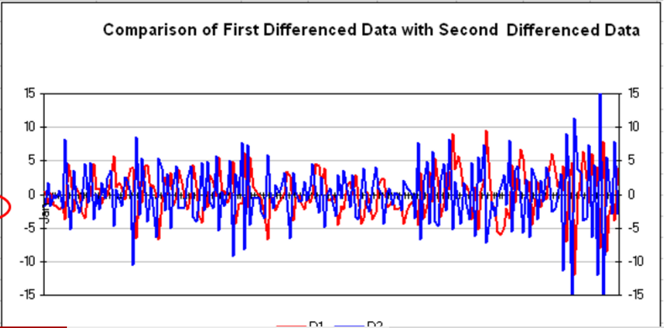


# Test for Stationarity

- Estimated regression for $D_{2,t} = \alpha + \beta D_{1,t-1}$.
- D-F is -12.948, which is the $t$ ratio on the slope parameter $\beta$.
- See the residuals oscillate about a mean of zero, no trend in either series.
- Intercept is 0.121 or about zero, so the mean is more likely to be constant.

OLS Regression Statistics

| | | |
|---|---|---|
| F-test | 167.647 | Prob(F) |
| MSE $^{1/2}$ | 2.449 | CV Regr |
| $R^2$ | 0.456 | Durbin-W |
| RBar $^2$ | 0.453 | Rho |
| Akaike Inf | 1.791 | Goldfeld- |
| Schwarz I | 1.808 | |
| 95% | Intercept | D1 |
| Beta | 0.121 | **-0.500** |
| S.E. | 0.172 | 0.039 |
| t-test | 0.700 | -12.948 |
| Prob(t) | 0.485 | 0.000 |
| Elasticity at Mean | | -0.012 |
| Variance Inflation F | | NA |
| Partial Correlation | | NA |
| Semipartial Correlat | | NA |



# DF Stationarity Test in Simetar

Dickey-Fuller (DF) function in *Simetar*
= DF ( Data Series, Trend, No. of Lags, No. of Diff to Test)

where:

- *Data Series* is the location of the data.
- *Trend* is "False" for the test described in the previous slides.
- *No. of Lags* is zero for the the test described in the previous slides.
- *No. of Diff* is the number of differences to test.

| | V | W | X | Y | Z | AA | AB | A |
|---|---|---|---|---|---|---|---|---|
| 1 | Dickey-Fuller Test assuming no trend and 0 lags | | | | | | | |
| 2 | No. Diff | **Trend** | **Lags** | DF Test Statistic | | | | |
| 3 | 0 | FALSE | 0 | **-1.868** | =DF($C$9:$C$212,W3,X3,V3) | | | |
| 4 | 1 | FALSE | 0 | **-12.948** | =DF($C$9:$C$212,W4,X4,V4) | | | |
| 5 | 2 | FALSE | 0 | **-24.967** | =DF($C$9:$C$212,W5,X5,V5) | | | |
| 6 | | | | | | | | |
| 7 | 0 | TRUE | 0 | **-1.952** | =DF($C$9:$C$212,W7,X7,V7) | | | |
| 8 | 1 | TRUE | 0 | **-12.916** | =DF($C$9:$C$212,W8,X8,V8) | | | |
| 9 | 2 | TRUE | 0 | **-24.903** | =DF($C$9:$C$212,W9,X9,V9) | | | |

# Summarize Stationarity

- $Y_t$ is the original data series.
- $D_{i,t}$ is the $i^{th}$ difference of the $Y_t$ series.
- We difference the data to make it stationary to guarantee the assumption that both mean and variance are constant.
- Dickey-Fuller test to determine the no. of differences needed to make series stationary.
  =DF(Data range, False, 0, No. of Differences)
- Test as many differences as necessary with and without trend and zero lags using =DF().
- Select the lowest number of differences with a DF test statistic more negative than -2.90 for the purpose of estimating the AR model (described next).

# NEXT: Determine the Number of Lags in the AR model

- Number of Lags, $q$, is the number of lagged values on the right-hand-side of the *OLS* equation.
- If the series is stationary with 1 difference, estimate the *OLS* model

$$D_{1,t} = \alpha + \beta_1 D_{1,t-1} + \beta_2 D_{1,t-2} + \dots$$

- The only question that remains is how many lags ($q$) of $D_{1,t}$ will we need to forecast the series.
- To determine the number of lags we use several tests.

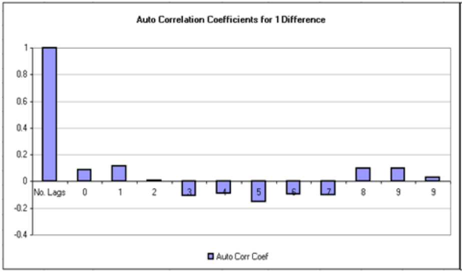
# Determining No. of Lags (Method #1)

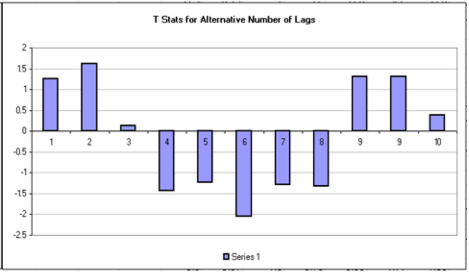
- Build a Sample Autocorrelation Table (SAC) =AUTOCORR(Data Series, No. Lags, No. Diff)
- Pick best no. of lags based on the last lag with a statistically significant $t$ value.

| | V | W | X | Y | Z | AA | AB | AC | AD |
|---|---|---|---|---|---|---|---|---|---|
| 7 | Sample Autocorrelation Coefficient Table to test for the best number of Lags | | | | | | | | |
| 8 | No. Diff | No. Lags | Auto Corr Coef | t Statistic | SE. Est. | Formula for Autocorr() Function | | | |
| 9 | 1 | 0 | 1 | | | | | | |
| 10 | 1 | 1 | 0.08781 | 1.2511223 | 0.07019 | =AUTOCORR($C$9:$C$212,W10,V10) | | | |
| 11 | 1 | 2 | 0.11502 | 1.6263217 | 0.07073 | =AUTOCORR($C$9:$C$212,W11,V11) | | | |
| 12 | 1 | 3 | 0.00875 | 0.1221616 | 0.07164 | =AUTOCORR($C$9:$C$212,W12,V12) | | | |
| 13 | 1 | 4 | -0.10255 | -1.431334 | 0.07165 | =AUTOCORR($C$9:$C$212,W13,V13) | | | |
| 14 | 1 | 5 | -0.08893 | -1.228838 | 0.07237 | =AUTOCORR($C$9:$C$212,W14,V14) | | | |
| 15 | 1 | 6 | -0.14881 | -2.041179 | 0.0729 | =AUTOCORR($C$9:$C$212,W15,V15) | | | |
| 16 | 1 | 7 | -0.09578 | -1.287613 | 0.07438 | =AUTOCORR($C$9:$C$212,W16,V16) | | | |
| 17 | 1 | 8 | -0.09946 | -1.326378 | 0.07499 | =AUTOCORR($C$9:$C$212,W17,V17) | | | |
| 18 | 1 | 9 | 0.09946 | 1.3149739 | 0.07564 | =AUTOCORR($C$9:$C$212,W18,V18) | | | |
| 19 | 1 | 9 | 0.09946 | 1.3149739 | 0.07564 | =AUTOCORR($C$9:$C$212,W19,V19) | | | |
| 20 | 1 | 10 | 0.02987 | 0.3915373 | 0.07628 | =AUTOCORR($C$9:$C$212,W20,V20) | | | |

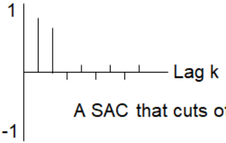
# Number of Lags for Time Series Model

- Bar chart of autocorrelation coefficients in Sample *AUTOCORR*() Table.
- The explanatory power of the distant lags is not large enough to warrant including in the model, based on their t stats, so do not include them.

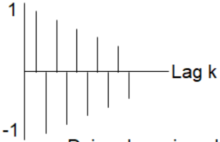


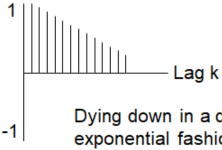
# Autocorrelation Charts of Sample Autocorrelation Coefficients (SAC)



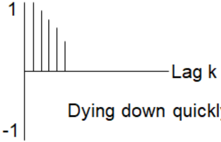

A SAC that cuts off



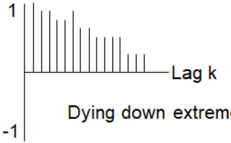

Dying down in a dampened exponential fashion – oscillation




Dying down in a dampened exponential fashion – no oscillation




Dying down quickly




Dying down extremely slowly

# Determining the Number of Lags (Method #2)

- Use Schwarz Information Criterion (SIC) for an information-theoretic determination of the best number of lags
- Find the number of lags which minimizes the *SIC*.
- In Simetar use the *ARLAG*() function which returns the optimal number of lags based on *SIC* test
  =ARLAG(Data Series, Constant, No. of Differences)

| | F | G | H | I | J | K | L | M |
|---|---|---|---|---|---|---|---|---|
| 34 | **Test for the Number of Lags based on the Schwarz Criteria.** | | | | | | | |
| 35 | =ARLAG(Range Raw Data, Constant, No. of Differences) | | | | | | | |
| 36 | No. Differences | | Yes Constant | | | | | |
| 37 | 1 | | 1 | | =ARLAG($B$9:$B$212,TRUE,F37) | | | |
| 38 | | | No Constant | | | | | |
| 39 | 1 | | 1 | | =ARLAG($B$9:$B$212,FALSE,F39) | | | |

# Number of Lags for AR(p,q) (Method #3)

- Partial autocorrelation coefficients used to estimate number of lags for $D_{i,t}$ in model.
- If $D_{1,t}$ is stationary then, define $D^*_{1,t} = D_{1,t} - \bar{D}_1$:
- Test for one lag use $\beta_1$ from $OLS$ regression model

$$D^*_{1,t} = \boldsymbol{\beta_1} D^*_{1,t-1} + e_t$$

- Test for two lags use $\beta_2$ from $OLS$ regression model

$$D^*_{1,t} = \beta_1 D^*_{1,t-1} + \boldsymbol{\beta_2} D^*_{1,t-2} + e_t$$

- Test for three lags use $\beta_3$ from $OLS$ regression model

$$D^*_{1,t} = \beta_1 D^*_{1,t-1} + \beta_2 D^*_{1,t-2} + \boldsymbol{\beta_3} D^*_{1,t-3} + e_t$$

- After each regression we only use the beta ($\beta_i$) for the last lagged term, i.e., the bold ones above. Use the $t$ test on the last $\beta_i$ to determine contribution of the last lag to explaining $D^*_{1,t}$.

# Note: Partial vs. Sample Autocorrelation

- Partial autocorrelation coefficients (PAC) show the contribution of adding one more lag (PAUTOCORR).
    - It takes into consideration the impacts of lower order lags.
    - A $\beta$ for the $3^{rd}$ lag shows the contribution of $3^{rd}$ lag after having lags 1-2 in place.

$$D^*_{1,t} = \beta_1 D^*_{1,t-1} + \beta_2 D^*_{1,t-2} + \boldsymbol{\beta_3} D^*_{1,t-3} + e_t$$

- Sample autocorrelation coefficients (SAC) show contribution of adding a particular lag (AUTOCORR).
    - A SAC for 3 lags shows the contribution of just the 3rd lag.

$$D^*_{1,t} = \beta D^*_{1,t-3} + e_t$$

- Thus the SAC does not equal the PAC.

# Number of Lags for Time Series Model

- Some authors suggest using SAC to determine the number of differences to achieve stationarity.
- If the SAC cuts off or dies down rapidly it is an indicator that the series is stationary.
- If the SAC dies down very slowly, the series is not stationary.
- This is a good check of the DF test, but we will rely on the DF test for stationarity.

# Summarize Stationarity/Lag Determination

- Make the data series stationary by differencing the data.
  - Use the Dickey-Fuller Test (DF $<$ -2.90) to find how many differences necessary to make the data stationary ($p$).
  - Use the =DF() function in Simetar.
- Use the sample autocorrelation coefficients (SACs) to determine how many lags ($q$) to include in the AR model.

  =AUTOCORR() function in Simetar

  (array formula!)
- Or... minimize the Schwarz Information Criterion to determine the number of lags ($q$) to include.

  =ARLAG() or =ARSCHWARZ() functions in Simetar