Agribusiness Analysis and Forecasting Value-at-risk and portfolio analysis

Texas A&M University

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- A model of price evolution
- Value-at-risk
- Portfolio analysis

We want a model of price evolution that accommodates:

- Price being non-stationary
- Prices can't be negative
- Probability distribution with minimal skew and kurtosis

A common basic assumption is that the expected return and variability of returns for many assets should be proportional to the price level.

This gives rise to the *geometric Brownian motion* (GBM):

$$dX = \mu X \, dt + \sigma X \, dW$$

Components of the GBM:

- *dX* is an infinitesimally small change in the price
- μ is the *annualized* excpected return
- σ is the *annualized* standard deviation of returns
- *dW* is an infinitesimally small increment to a *Wiener process*
- *dt* is an infinitesimally small change in time *measured in years*

Components of the process



Applying *Ito's Lemma*, and using the properties of the Weiner process... Discrete time version of the of the process for price *returns* (In changes):

$$\ln(x_t) - \ln(x_{t-\Delta t}) = \Delta \ln x_t = \left(\mu - \frac{\sigma^2}{2}\right) \Delta t + \sigma z_t \sqrt{\Delta t}$$

Here, z_t is a standard normal draw

What is Δt ?



Relatively large value of Δt

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What is Δt ?



Smaller value of Δt

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What is Δt ?



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Data wrangling

- Gather historical price observations
- Convert the prices to In changes:

$$R_t = \ln(X_t) - \ln(X_{t-1})$$

or

$$R_t = ln\left(rac{X_t}{X_{t-1}}
ight)$$

 MorningStar or Yahoo provides stock prices on web, downloadable in Excel format

Calibrating μ and σ from historical data

A simple moment matching approach involves calculating the mean change in ln(X), recovering the residuals (w_t) , and then calculating the annualized the parameters:

$$R_t = \beta_0 + w_t$$

- Calculate \hat{eta}_0 as the mean of R_t
- Calculate $\hat{\sigma}_W$ as the standard deviation of the residuals

Calibrating μ and σ from historical data

Next, use the estimated $\hat{\beta}_0$ and $\hat{\sigma}_W$ to calculate the values for the annualized parameters:

$$\sigma = \frac{\hat{\sigma}_W}{\sqrt{\Delta t}}$$

$$u - \frac{\sigma^2}{2} = \frac{\hat{\beta}_0}{\Delta t}$$

Here, Δt reflects the observation frequency of the historical data (this need not match the Δt you will use in a simulation)



Amazon stock price

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- Define and describe VaR.
- How to calculate VaR.
- Apply these methods to calculating VaR for a single stock and portfolio.
- Use VaR for a farm simulation model.

- One of the popular and traditional measures of risk is volatility.
- A problem with volatility is that it reflects both downside and *upside* risk:
 - A particular stock can be volatile because it suddenly jumps higher.
 - Usually, investors are not distressed by gains.
- For most investors, "risk" is about the danger of *losing* money, and VaR reflects this

- VaR answers the question, "What is my worst-case scenario?" or How much could I lose in a really bad month?"
- Intuitive definition: VAR summarizes the worst loss over a target horizon with a given level of confidence
- VAR defines a particular quantile of the projected probability distribution of gains and losses over the target horizon.

- VaR has three components: a time period, a confidence level and a loss amount (or loss percentage)
- Example: What is the *dollar amount* that I have a 5% *chance* of losing more than over the next *one week*?
- As we see, VaR has three elements: a relatively high level of confidence (5% or 1%), a time period (e.g., day, week, month, year), and an estimate of loss (in dollar or percentage terms).



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Since the time period is a variable, different calculations may specify different time periods - there is no "correct" time period.

- Commercial banks or brokerages typically calculate a daily VaR, asking themselves how much they can lose in a day.
- Pension funds, on the other hand, might calculate a monthly VaR.
- In farm simulation model we might use a one-year VaR.

Value at Risk (VaR)

- On selecting the 'c' value it is common to use the 5% level.
- This is to say we want to know the value of returns which we will exceed 95% of the time.
- If simulating 1000 iterations of the change in portfolio value, the quantile will be the 50th value, so we can sort the stochastic results and read the 50th value.

Value at Risk (VaR)

If 'c' is the selected <u>confidence level</u>, VaR corresponds to the <u>1-c lower tail</u> of the probability distribution (the quantile).



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Calculating VaR using simulation

- Calculate the annualized return(s) for the price(s)
- Calculate annualized volatility(ies)
- Generate a standard normal draw (or multivariate/joint standard normal draws).
- Select the time horizon. For example, to calculate a one-month VaR, $\Delta t = 1/12 = 0.083$
- Once you have all the parameters, calculate returns under the standard process
- Use returns to calculate price levels: $S_1 = S_0 \times e^{((\mu - 0.5\sigma^2)\Delta t + \sigma z \sqrt{\Delta t})}$
- Calculate change in portfolio value
- Calcaulte VaR: with 500 simulated changes in port. value in ascending order, the 5% VaR is the 25th value on the second secon

VaR in Farm Simulation

- Consider VaR to be based on the change in ending wealth or cash
- VaR is the equity capital that should be set aside to cover most all potential losses with a probability of c.
- Thus, the VaR is the amount of capital reserves that should be held to meet shortfalls.



VaR for Comparing Risky Alternatives

Simulate multiple scenarios and calculate VAR for each alternative.



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Amazon stock price VaR; portfolio VaR

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- We can use simulation to pick a portfolio of investments
- Many business decisions can be couched in a portfolio analysis framework
- A portfolio can represent any set of risky alternatives the decision maker considers:
 - Stocks, mutual funds, bond mix
 - Crop insurance levels to purchase
 - Mix of crops to plant

- Basis for portfolio analysis overall risk can be reduced by investing in two or more risky instruments rather than one
- If the correlation between the risky investments is negative $\rho_{x,y} < 0$ this is called *hedging*
- Portfolio variance can still be reduced even if there is positive correlation among the prices/returns. This is investment *diversification*

Application to business – given two business activiites, we may consider a combination of the two rather than specializing in only one

- Mid West farmers traditionally raised corn and fed cattle and hogs, now they raise corn and soybeans.
- Grow cotton and/or alfalfa
- An undiversified portfolio might be to grow only corn.

There are thousands of available investments; which ones to include in the portfolio?

- Own stocks in IBM and Microsoft
- A portfolio may include Apple, NetFlix, Amazon, Facebook, ...
- You can apply simulation method to find an "optimal" portion of investment in each risky asset

- Select investment instruments (e.g., stocks) to analyze
- Gather returns historical data
- Simulate stochastic returns (the Rs) for all instruments
- Olice a weighted return for each portfolio
- Calculate the distributions of changes in portfolio value for each candidate portfolio
- Use some criterion to choose a portfolio Alternative approach: keep everything in terms of returns rather than dollars

- Example: portfolio analysis with six risky assets.
- Find the best combination of the assets
- In many cases the risky assets move together (positive correlation coefficients)

Linear (Correlation M	latrix				
	Inv 1	Inv 2	Inv 3	Inv 4	Inv 5	Inv 6
Inv 1	1	-0.11	-0.19	0.50	0.29	-0.37
Inv 2		1	0.13	-0.10	0.42	-0.43
Inv 3			1	0.35	-0.54	-0.14
Inv 4				1	0.32	-0.35
Inv 5					1	-0.44
Inv 6						1

Ten portfolios analyzed and expressed as proportion of the total investment for each of the six risky assets

Portfol	ios Expressed	as Fraction	s of Money	/ Invested	in Each Inv	estment A	Iternative
	Inv 1	Inv 2	Inv 3	Inv 4	Inv 5	Inv 6	Check
P1	1						1
P2		1					1
P3			1				1
P4				1			1
P5					1		1
P6						1	1
P7	0.166667	0.1666667	0.166667	0.166667	0.166667	0.166667	1
P8	0.3	0.3			0.3	0.1	1
P9		0.3	0.3	0.1		0.3	1
P10			0.3	0.3	0.3	0.1	1

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- The statistics below for 10 simulated portfolios show variance reduction relative to investing exclusively in one instrument.
- Look at the CVs across Portfolios P1-P10 (in this context, CV is similar to the inverse of *risk-adjusted return*)
- Portfolios 1-5 are 100% in one instrument.
- Portfolios 6-10 are combinations, note the reductions in CVs.

Iteration	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Mean	77.1	134.4	164.5	133.7	99.3	103.4	118.7	103.6	134.1	129.6
StDev	181.0	160.6	152.5	138.5	177.2	128.7	58.9	97.7	66.9	75.4
CV	234.7	119.6	92.7	103.6	178.5	124.5	49.6	94.3	49.9	58.2
Min	-403.1	-350.9	-245.0	-264.9	-495.5	-281.1	-33.6	-187.7	-34.7	-79.5
Max	606.9	587.0	747.1	554.5	659.9	494.9	299.0	391.2	408.1	358.7



Portfolio analysis

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